In Fig. 1, results of calculation from Eq. (18) for the velocity of isotherm propagation are compared with experimental data. As is evident from Fig. 1, the discrepancy is no more than 2-5%, which indicates that the assumptions made above are valid. The use of Eq. (18) with constant G_v offers the possibility of determining the heating of the zone H in the form

$$H = \frac{\Delta H}{\tau_0}\tau, \tag{19}$$

where $\Delta H/\tau_0$ is the velocity of isotherm propagation.

NOTATION

P, pressure, N/m^2; T, temperature, °K; σ_v, vapor saturation of medium, m^3/m^3; σ_w, saturation of medium with water; r_p, heat of phase transition, J/kg; ε, porosity; c_p^c, specific heat of sand, J/kg·K; c_p^w, specific heat of medium, J/kg·K; c_p^v, specific heat of vapor; G_v, vapor flow rate, $kg/m^2\cdot sec$; ρ_v, vapor density, kg/m^3; ρ_w, water density, kg/m^3; v_v, specific volume of vapor, m^{-3}; g_p, flux-density of phase-transition material; $kg/m^3\cdot sec$; τ, time, sec; μ, molecular weight; τ_0, heating time of condensate up to T; v, rate of growth of heated zone, m/sec.

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A THEORY OF BINARY LIQUID MIXTURE EVAPORATION FROM CAPILLARIES

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A theory of binary liquid mixture evaporation from capillaries is developed. Expressions defining mixture composition and the law of meniscus motion during evaporation are obtained.

We will consider isothermal evaporation of binary mixtures from capillaries. Because of the brevity of the diffusion relaxation time in the gas phase as compared to the evaporation time, the component vapor flows will be regarded as steady state for any position of the moving meniscus. It is assumed that molecules of each mixture component can experience a phase transition on the liquid–gas boundary. The gas mixture into which evaporation occurs consists of molecules of the volatile mixture components and molecules of a gas which is chemically inert with respect to the components. The liquid surface is assumed impermeable to vapor–gas mixture molecules.

In the general case, during evaporation of liquid mixtures there is a constant change in mixture composition due to the differing volatilities of the components. We will assume that the volumes occupied by the components are additive:

$$V = \frac{M_1}{\rho_1^0} + \frac{M_2}{\rho_2^0}. \tag{1}$$

The content of the first component is defined by the law of conservation of mass in the form

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$$-\frac{dM_1}{dt} = m_1 G_1 S. \tag{2}$$

The change in mass of the second component is determined by the equation of total mass balance

$$-\frac{dM}{dt} = m_1 G_1 S + m_2 G_2 S. \tag{3}$$

At an arbitrary time the masses M_1 and M can be represented in the form

$$M_1 = \rho_1 (L - l)\, S, \tag{4}$$

$$M = \rho (L - l)\, S. \tag{5}$$

The relationship between ρ_1 and ρ_2 can be formulated from Eqs. (1), (4), (5) in the following manner:

$$\frac{\rho_1}{\rho_1^0} + \frac{\rho_2}{\rho_2^0} = 1. \tag{6}$$

Then the solution density can be related to the varying density of one of the components:

$$\rho = \rho_1 + \rho_2^0 \left(1 - \frac{\rho_1}{\rho_1^0}\right). \tag{7}$$

With consideration of Eq. (7), we transform system (2)-(5) and obtain equations for the density of one of the mixture components as a function of meniscus coordinate, and also the law for motion of the interphase boundary:

$$(l - L)\frac{d\rho_1}{dl} + \rho_1 = \frac{K_1 \rho_2^0}{1 + K_1 \left(\frac{\rho_2^0}{\rho_1^0} - 1\right)}, \tag{8}$$

$$\frac{dl}{dt} = \frac{G_1 m_1 + G_2 m_2}{\rho_2^0} \left[1 + K_1 \left(\frac{\rho_2^0}{\rho_1^0} - 1\right)\right], \tag{9}$$

where

$$K_1 = \frac{G_1 m_1}{G_1 m_1 + G_2 m_2}.$$

Equations (8), (9) can be integrated in quadratures with the assumption that at any moment in time a homogeneous component density distribution over capillary length is established:

$$\int\limits_{\rho_1}^{\rho_{10}} \frac{\left[1 + K_1 \left(\frac{\rho_2^0}{\rho_1^0} - 1\right)\right] d\rho_1}{K_1 \rho_2^0 - \rho_1 - K_1 \rho_1 \left(\frac{\rho_2^0}{\rho_1^0} - 1\right)} = \ln\left(\frac{L}{L - l}\right), \tag{10}$$

$$t = \rho_2^0 \int\limits_0^l \frac{dl}{(G_1 m_1 + G_2 m_2)\left[1 + K_1 \left(\frac{\rho_2^0}{\rho_1^0} - 1\right)\right]}. \tag{11}$$

Equations (10), (11) allow determination of the binary mixture liquid phase composition and the law of meniscus motion during evaporation. The applicability of these equations is not limited to any particular evaporation regime, and the values of the vapor flux densities G_1 and G_2 can be specified from the conditions of the particular problem.

We will consider the case where the pressure of the gas mixture in the capillary is constant, which corresponds to Stefan flow of the vapor molecules. The quantities G_1 and G_2 can be found using the system of equations [1]

$$\frac{G_2 c_1 - G_1 c_2}{n D_{12}} - \frac{G_1 c_3}{n D_{13}} = -\frac{dc_1}{dx}, \tag{12}$$

$$\frac{G_1 c_2 - G_2 c_1}{n D_{12}} - \frac{G_2 c_3}{n D_{23}} = -\frac{dc_2}{dx}, \tag{13}$$

$$\frac{c_3}{n}\left(\frac{G_1}{D_{13}}+\frac{G_2}{D_{23}}\right)=-\frac{dc_3}{dx}. \tag{14}$$

Boundary conditions are taken as follows:

$$c_1(0)=c_{10},\quad c_1(l)=c_{1s}, \tag{15}$$

$$c_2(0)=c_{20},\quad c_2(l)=c_{2s}. \tag{16}$$

The relative concentrations of molecules in the vapor–gas mixture are interrelated by the condition

$$c_1+c_2+c_3=1. \tag{17}$$

Then, integrating Eqs. (12)-(14) with consideration of Eqs. (15)-(17), we obtain a system of equations defining G_1 and G_2:

$$\frac{G_1 l}{G_1 l+nD_{23}\ln\left(\frac{c_{30}}{c_{3l}}\right)-\left(\frac{D_{23}}{D_{13}}\right)G_1 l}+\left[c_{10}-\frac{G_1 l}{G_1 l\left(1-\frac{D_{23}}{D_{13}}\right)+nD_{23}\ln\left(\frac{c_{30}}{c_{3l}}\right)}-\right.$$

$$\left.-\frac{c_{30}D_{12}G_1 l\left(\frac{1}{D_{13}}-\frac{1}{D_{12}}\right)}{G_1 l\left(1-\frac{D_{23}}{D_{13}}\right)+n(D_{13}-D_{12})\ln\left(\frac{c_{30}}{c_{3l}}\right)}\right]\exp\left[-\frac{D_{23}}{D_{12}}\ln\left(\frac{c_{30}}{c_{3l}}\right)-\frac{G_1 l}{nD_{12}}\left(1-\frac{D_{23}}{D_{13}}\right)\right]+$$

$$+\frac{D_{12}c_{3l}G_1 l\left(\frac{1}{D_{13}}-\frac{1}{D_{12}}\right)}{G_1 l\left(1-\frac{D_{23}}{D_{13}}\right)+n(D_{23}-D_{12})\ln\left(\frac{c_{30}}{c_{3l}}\right)}-c_{1s}=0, \tag{18}$$

$$G_2=\frac{nD_{23}}{l}\ln\left(\frac{c_{30}}{c_{3l}}\right)-\frac{D_{23}}{D_{13}}G_1. \tag{19}$$

The equations obtained take on their simplest form in the case where binary diffusion coefficients have similar values, i.e., $D_{12}\approx D_{13}\approx D_{23}\approx D$. In this case

$$G_1=\left(\frac{nD}{l}\right)\frac{c_{1s}c_{30}-c_{10}c_{3l}}{c_{30}-c_{3l}}\ln\frac{c_{30}}{c_{3l}}. \tag{20}$$

$$G_2=\left(\frac{nD}{l}\right)\frac{c_{2s}c_{30}-c_{20}c_{3l}}{c_{30}-c_{3l}}\ln\frac{c_{30}}{c_{3l}} \tag{21}$$

It can be shown that the quantities G_1 and G_2 have a similar form when true diffusion coefficients are replaced by effective ones [2, 3]. In the case where $c_{1s}+c_{2s}\ll 1$ Eqs. (18), (19) transform to the corresponding expressions obtained from Fick's law. If one of the relative concentrations of the mixture components is taken equal to zero (for example, c_2), then Eq. (18) transforms to the well known case of evaporation of a pure liquid [4].

During evaporation of liquid mixtures change in component concentration within the solution can occur, which leads in turn to change in the saturated vapor pressure above the meniscus. The relative vapor concentrations above an ideal solution follow Raul's law:

$$c_{1s}=c_{1s}^{0}N_1=c_{1s}^{0}\frac{\frac{\rho_1}{\mu_1}}{\frac{\rho_1}{\mu_1}+\frac{\rho_2}{\mu_2}}, \tag{22}$$

$$c_{2s}=c_{2s}^{0}N_2=c_{2s}^{0}\frac{\frac{\rho_2}{\mu_2}}{\frac{\rho_1}{\mu_1}+\frac{\rho_2}{\mu_2}}. \tag{23}$$

We now substitute Eqs. (20), (21) in Eq. (8) and solve the same with consideration of Eqs. (7), (22), (23):

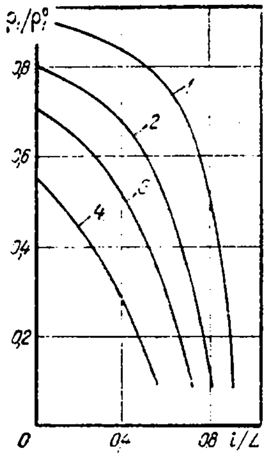

Fig. 1.

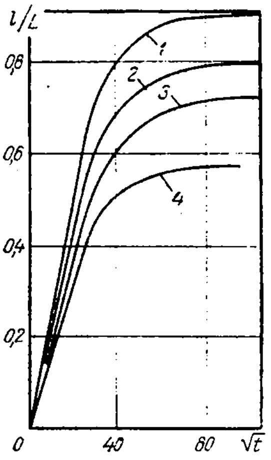

Fig. 2.

Fig. 1. Change in density of more volatile component versus dimensionless capillary length during mixture evaporation: 1) $\rho_{10}/\rho_1^0 = 0.9$; 2) 0.8; 3) 0.7; 4) 0.55.

Fig. 2. Meniscus coordinate versus time (1-4, see Fig. 1); $\sqrt{t}$, sec.

$$\left(\frac{1-\frac{\rho_1}{\rho_1^0}}{1-\frac{\rho_{10}}{\rho_1^0}}\right)^{1+\gamma}\left(\frac{\rho_{10}}{\rho_1}\right)^{\gamma}=\frac{1}{1-\frac{l}{L}} \tag{24}$$

where $\gamma = c_{2s}^0/(c_{1s}^0 - c_{2s}^0)$. In writing the last equation, a simplifying assumption was made, which does not limit generality of the results, that the mixture vapor content in the surrounding gas is negligibly small, i.e., $c_{10} = c_{20} = 0$.

Equation (24) can be used to establish the relationship between the density of one of the mixture components as it varies during evaporation and the meniscus coordinate. As an example, Fig. 1 shows the curve $\rho_1(l)$ in dimensionless form, obtained by numerical solution of Eq. (24). It was assumed that the more volatile component would be studied ($\gamma = 0.01$).

The expressions obtained permit use of Eq. (11) to determine the meniscus motion law $l(t)$. Figure 2 shows relative meniscus coordinate versus time for the cases considered in Fig. 1. The calculations were performed for equal binary diffusion coefficients ($D = 0.1$ cm^2/sec) at $T = 323$°K for $\rho_1^0 = \rho_2^0 = 0.7\cdot10^3$ (kg/m^3), $\mu_2 = 200$ (kg/mole), $\mu_1 = 100$ (kg/mole).

As follows from Fig. 2, evaporation does not follow the Stefan law in these cases. However, the speed of meniscus motion is less than the evaporation rate of the more volatile component in pure form, which is caused by a reduction in concentration of vapor of the latter above the surface due to the presence of the second component. As the first component continues to evaporate there is a sharp reduction in the speed of the process, which tends asymptotically to the corresponding value for the pure second, less volatile, component.

With consideration of conditions (22), (23), Eqs. (8), (9), obtained above, transform to the corresponding expressions of [5], which were obtained by using Fick's laws to study evaporation of binary mixtures with one component practically nonvolatile. As was shown in [5], the difference between theoretical and experimental data does not exceed 4%.

NOTATION

V, liquid phase volume, m^3; M_1, M_2, masses of first and second components in liquid phase, kg; ρ_1^0, ρ_2^0, density of first and second components in pure form at specified temperature, kg/m^3; t, time, sec; m_1, m_2, molecular mass of first and second components, kg; G_1, G_2, molecular flow density of first and second components, 1/m$^2\cdot$sec; ρ_1, ρ_2, mean densities of components forming mixture at arbitrary time, kg/m^3; L, total capillary length, m; l, current meniscus coordinate, m; ρ_{10}, initial density of first component, kg/m^3; c_i, relative molecular concentration of vapor components (i = 1, 2) and external gas (i = 3) per unit volume of vapor gas mixture; D_{ij}, binary diffusion coefficient in vapor–gas phase, m^2/sec; x, spatial coordinate, m; c_{10}, c_{20}, relative molecular concentrations of vapor components in surrounding gas at initial time: c_{1s}, c_{2s}, relative concentrations of first and second components above meniscus; c_{30}, c_{3l}, relative molecular concentration of gas forming atmosphere into which evaporation occurs; c^0_{1s}, c^0_{2s}, relative molecular concentrations of vapors of first and second components for pure liquids; N_1, N_2, molar fractions of first and second components; μ_1 and μ_2, molar masses for first and second components, kg/mole.

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EFFECT OF ICE SURFACE ORIENTATION ON INTENSITY OF WATER-TO-ICE HEAT TRANSFER UNDER FREE CONVECTION CONDITIONS

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A graphical construction permitting determination of the heat-transfer coefficient for various slopes of an ice surface is proposed.

Heat transfer between water and frozen ground is the main factor causing damage to water reservoirs in regions where the ground does not freeze for years at a time. To predict failure of retaining dams it is necessary to have data characterizing the intensity of thermal processes on the water-frozen soil boundary, in particular, heat-transfer coefficient values.

The profile of the retaining wall may vary greatly, so to simplify calculations it can be approximated by a collection of individual inclined segments. The problem of thermal calculation then reduces to determination of the heat transfer from the water to an inclined plane surface having a negative temperature. In the process of thermal interaction with frozen ground, melting of ice occurs with subsequent removal of the water formed due to a density difference, i.e., the aggregate state of the ice changes and the liquid phase thus produced is removed under the action of free convection.

At the present time the Soviet and foreign literature provides a number of studies of heat transfer to inclined plane surfaces [1-4], but in those studies heat transfer took place with no change in aggregate state of the material. Without considering this factor results in elevated results [5]. Nor is it possible to use results from studies of ice melting with ice specimens in the form of spheres [5-7], cylinders [5], cubes [8], or horizontal surfaces [9, 10], since under free convection conditions the form of the surface has a great effect on heat transfer [1]. Only for the rarely found case of a perpendicular retaining wall can data on heat transfer between water and a vertical ice plate [11-14] be used with assurance.

In studying heat transfer on a water–ice boundary, aside from change in the aggregate state of the ice, another factor producing difficulty is the change in water density with

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